



**MAA AMC**  
*American Mathematics Competitions*

**MAA American Mathematics Competitions**

**34<sup>th</sup> Annual**

**AMC 8**

**American Mathematics Competition 8**

**Tuesday, November 13, 2018**

## **INSTRUCTIONS**

1. **DO NOT OPEN THIS BOOKLET UNTIL YOUR COMPETITION MANAGER TELLS YOU.**
2. This is a 25-question multiple-choice exam. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers that are properly marked on the answer form will be scored.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are **NOT** allowed. No problems on the exam *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the exam, your competition manager will ask you to record your name and other information on the answer sheet.
8. You will have 40 minutes to complete the exam once your competition manager tells you to begin.
9. When you finish the exam, *sign your name* in the space provided at the bottom of the answer sheet.

The MAA Committee on the American Mathematics Competitions reserves the right to disqualify scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.



# MAA AMC

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## AMC 8

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### MAA Partner Organizations

We acknowledge the generosity of the following organizations in supporting the MAA AMC and

Invitational Competitions:

#### **Patron's Circle**

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#### **Innovator's Circle**

The D. E. Shaw Group

Two Sigma

#### **Winner's Circle**

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#### **Collaborator's Circle**

American Statistical Association

Casualty Actuarial Society

Conference Board of the Mathematical Sciences

Mu Alpha Theta

Society for Industrial and Applied Mathematics

1. An amusement park has a collection of scale models, with ratio 1:20, of buildings and other sights from around the country. The height of the United States Capitol is 289 feet. What is the height in feet of its replica at this park, rounded to the nearest whole number?

(A) 14      (B) 15      (C) 16      (D) 18      (E) 20

2. What is the value of the product

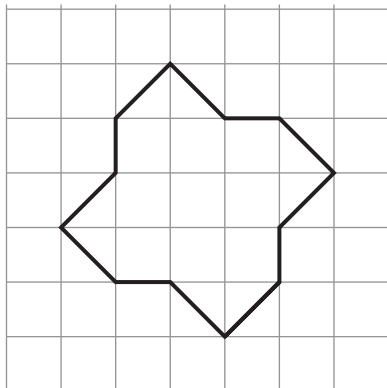
$$\left(1 + \frac{1}{1}\right) \cdot \left(1 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{3}\right) \cdot \left(1 + \frac{1}{4}\right) \cdot \left(1 + \frac{1}{5}\right) \cdot \left(1 + \frac{1}{6}\right)?$$

(A)  $\frac{7}{6}$       (B)  $\frac{4}{3}$       (C)  $\frac{7}{2}$       (D) 7      (E) 8

3. Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last one present in the circle?

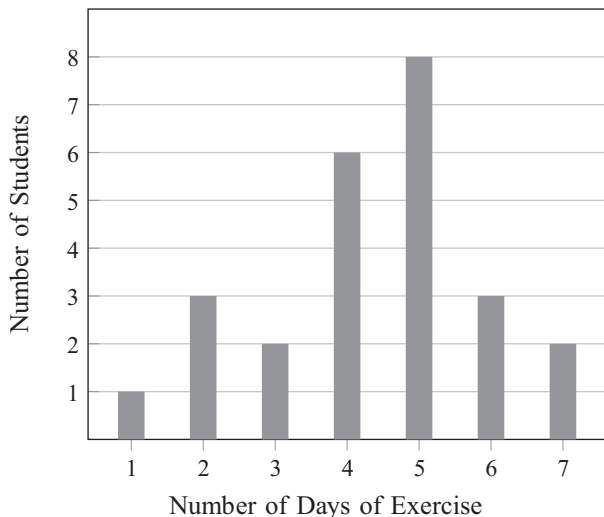
(A) Arn      (B) Bob      (C) Cyd      (D) Dan      (E) Eve

4. The twelve-sided figure shown has been drawn on 1 cm  $\times$  1 cm graph paper. What is the area of the figure in  $\text{cm}^2$ ?



(A) 12      (B) 12.5      (C) 13      (D) 13.5      (E) 14

5. What is the value of  $1+3+5+\cdots+2017+2019-2-4-6-\cdots-2016-2018$ ?
- (A)  $-1010$       (B)  $-1009$       (C)  $1008$       (D)  $1009$       (E)  $1010$
6. On a trip to the beach, Anh traveled 50 miles on the highway and 10 miles on a coastal access road. He drove three times as fast on the highway as on the coastal road. If Anh spent 30 minutes driving on the coastal road, how many minutes did his entire trip take?
- (A) 50      (B) 70      (C) 80      (D) 90      (E) 100
7. The 5-digit number  $\underline{2} \underline{0} \underline{1} \underline{8} \underline{U}$  is divisible by 9. What is the remainder when this number is divided by 8?
- (A) 1      (B) 3      (C) 5      (D) 6      (E) 7
8. Mr. Garcia asked the members of his health class how many days last week they exercised for at least 30 minutes. The results are summarized in the following bar graph, where the heights of the bars represent the number of students.

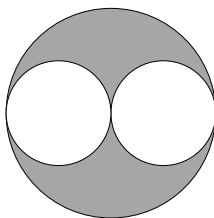


What was the mean number of days of exercise last week, rounded to the nearest hundredth, reported by the students in Mr. Garcia's class?

- (A) 3.50      (B) 3.57      (C) 4.36      (D) 4.50      (E) 5.00

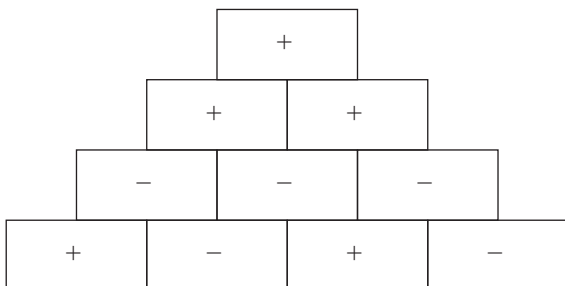
9. Tyler is tiling the floor of his 12 foot by 16 foot living room. He plans to place one-foot by one-foot square tiles to form a border along the edges of the room and to fill in the rest of the floor with two-foot by two-foot square tiles. How many tiles will he use?
- (A) 48      (B) 87      (C) 91      (D) 96      (E) 120
10. The *harmonic mean* of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?
- (A)  $\frac{3}{7}$       (B)  $\frac{7}{12}$       (C)  $\frac{12}{7}$       (D)  $\frac{7}{4}$       (E)  $\frac{7}{3}$
11. Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.
- |   |   |   |
|---|---|---|
| X | X | X |
| X | X | X |
- If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column?
- (A)  $\frac{1}{3}$       (B)  $\frac{2}{5}$       (C)  $\frac{7}{15}$       (D)  $\frac{1}{2}$       (E)  $\frac{2}{3}$
12. The clock in Sri's car, which is not accurate, gains time at a constant rate. One day as he begins shopping he notes that his car clock and his watch (which is accurate) both say 12:00 noon. When he is done shopping, his watch says 12:30 and his car clock says 12:35. Later that day, Sri loses his watch. He looks at his car clock and it says 7:00. What is the actual time?
- (A) 5:50      (B) 6:00      (C) 6:30      (D) 6:55      (E) 8:10
13. Laila took five math tests, each worth a maximum of 100 points. Laila's score on each test was an integer between 0 and 100, inclusive. Laila received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Laila's score on the last test?
- (A) 4      (B) 5      (C) 9      (D) 10      (E) 18

14. Let  $N$  be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of  $N$ ?
- (A) 15      (B) 16      (C) 17      (D) 18      (E) 20
15. In the diagram below, a diameter of each of the two smaller circles is a radius of the larger circle. If the two smaller circles have a combined area of 1 square unit, then what is the area of the shaded region, in square units?

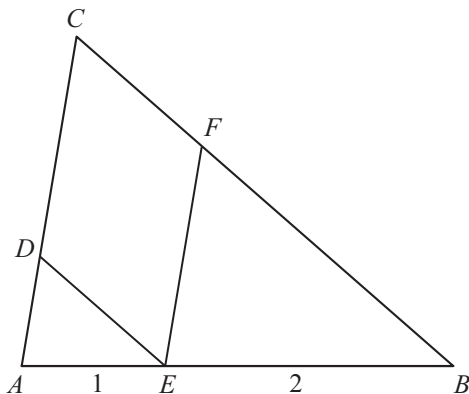


- (A)  $\frac{1}{4}$       (B)  $\frac{1}{3}$       (C)  $\frac{1}{2}$       (D) 1      (E)  $\frac{\pi}{2}$
16. Professor Chang has nine different language books lined up on a bookshelf: two Arabic, three German, and four Spanish. How many ways are there to arrange the nine books on the shelf keeping the Arabic books together and keeping the Spanish books together?
- (A) 1440      (B) 2880      (C) 5760      (D) 182,440      (E) 362,880
17. Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is 10,560 feet, and Bella covers  $2\frac{1}{2}$  feet with each step. How many steps will Bella take by the time she meets Ella?
- (A) 704      (B) 845      (C) 1056      (D) 1760      (E) 3520
18. How many positive factors does 23,232 have?
- (A) 9      (B) 12      (C) 28      (D) 36      (E) 42

19. In a sign pyramid a cell gets a “+” if the two cells below it have the same sign, and it gets a “−” if the two cells below it have different signs. The diagram below illustrates a sign pyramid with four levels. How many possible ways are there to fill the four cells in the bottom row to produce a “+” at the top of the pyramid?



- (A) 2      (B) 4      (C) 8      (D) 12      (E) 16
20. In  $\triangle ABC$ , point  $E$  is on  $\overline{AB}$  with  $AE = 1$  and  $EB = 2$ . Point  $D$  is on  $\overline{AC}$  so that  $\overline{DE} \parallel \overline{BC}$  and point  $F$  is on  $\overline{BC}$  so that  $\overline{EF} \parallel \overline{AC}$ . What is the ratio of the area of  $CDEF$  to the area of  $\triangle ABC$ ?

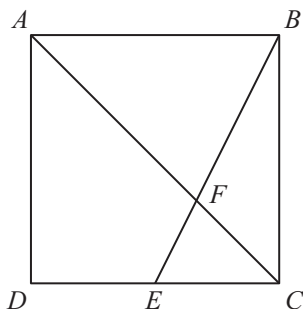


- (A)  $\frac{4}{9}$       (B)  $\frac{1}{2}$       (C)  $\frac{5}{9}$       (D)  $\frac{3}{5}$       (E)  $\frac{2}{3}$

21. How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

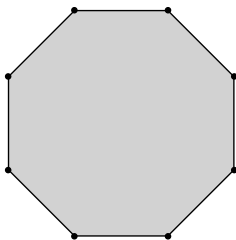
(A) 1      (B) 2      (C) 3      (D) 4      (E) 5

22. Point  $E$  is the midpoint of side  $\overline{CD}$  in square  $ABCD$ , and  $\overline{BE}$  meets diagonal  $\overline{AC}$  at  $F$ . The area of quadrilateral  $AFED$  is 45. What is the area of  $ABCD$ ?



(A) 100      (B) 108      (C) 120      (D) 135      (E) 144

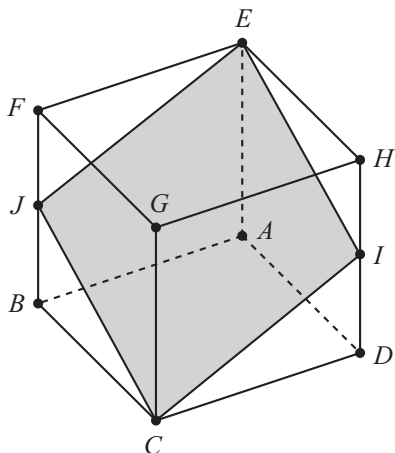
23. From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?



(A)  $\frac{2}{7}$       (B)  $\frac{5}{42}$       (C)  $\frac{11}{14}$       (D)  $\frac{5}{7}$       (E)  $\frac{6}{7}$



24. In the cube  $ABCDEFGH$  with opposite vertices  $C$  and  $E$ ,  $J$  and  $I$  are the midpoints of edges  $\overline{FB}$  and  $\overline{HD}$ , respectively. Let  $R$  be the ratio of the area of the cross-section  $EJCI$  to the area of one of the faces of the cube. What is  $R^2$ ?



- (A)  $\frac{5}{4}$       (B)  $\frac{4}{3}$       (C)  $\frac{3}{2}$       (D)  $\frac{25}{16}$       (E)  $\frac{9}{4}$
25. How many perfect cubes lie between  $2^8 + 1$  and  $2^{18} + 1$ , inclusive?
- (A) 4      (B) 9      (C) 10      (D) 57      (E) 58



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### **How will I receive my score?**

Scores and solutions will be sent to your competition manager who can share that information with you. You can use the solutions to learn more mathematics and enhance your problem-solving skills.

### **Are there more math competitions that I can participate in?**

The MAA American Mathematics Competitions also offers two high school level mathematics exams that are open to younger participants. These are both 25-question, 75-minute, multiple-choice exams designed to promote the development of problem-solving skills. For more information visit [maa.org/amc](http://maa.org/amc).

### **How can I prepare for future math competitions?**

The best way to prepare for the MAA American Mathematics Competitions is to practice creative, analytical thinking throughout the year. Schools involved with the MAA AMC often have year-round activities such as special classes, math clubs, or other extracurricular groups. Individual students can benefit greatly from practicing math problems from past MAA AMC exams.

### **Questions?**

Questions and comments about problems and solutions for this exam should be sent to:

[amchq@maa.org](mailto:amchq@maa.org)

Send questions and comments about administrative arrangements to:

[amcinfo@maa.org](mailto:amcinfo@maa.org)

or

MAA American Mathematics Competitions  
P.O. Box 471  
Annapolis Junction, MD 20701